Instructions: Work neatly. *Show at least one step of your work (where appropriate) for full credit.*

- You can have a 3" × 5" formula card with formulas only, no worked problems.
- **Study the quizzes.**
- **Study the homework and do similar problems to those in the homework for practice.**
- **Work lots of problems!**
- **Topical breakdown with example problems:**
  - critical numbers;
  - the closed interval method;
  - the Mean Value Theorem;
  - determining intervals on which a function is increasing and decreasing and use this information to perform the first derivative test;
  - determining the intervals on which a function is concave up and concave down and use this information to perform the second derivative test;
  - inflection points;
  - limits at infinity and horizontal asymptotes;
  - curve sketching;
  - optimization problems;
  - Newton's method;
  - Anti-derivatives;
  - Inverse functions and their derivatives.

  Warning: The above example problems are not an exhaustive list! Study the homework too.
- **Additional Information:** For curve sketching, it will be enough that you know how to sketch the graph of functions of the following form: (a) any cubic polynomial, (b) and any rational function of one of the following forms

$$\frac{ax^2 \pm b}{cx^2 \pm d}, \quad \frac{ax \pm b}{cx^2 \pm d}, \quad \frac{ax^2 \pm b}{cx \pm d}, \quad a, c > 0 \quad b, d \geq 0.$$

  Be able to label, when appropriate, x, y-intercepts, critical points, local maxima and minima, inflection points, and horizontal and vertical asymptotes.

SAMPLE EXAM 2 (does not contain anything past section 4)

1. (15 pts) Evaluate the following limits at infinity.
   (a) $\lim_{t\to\infty} \frac{2t^3-6t^2+1}{3t^3-5t+1} =$
   (b) $\lim_{t\to-\infty} \frac{2t^3-6t^2+1}{1-2t^2} =$
   (c) $\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+2x+2}} =$

2. (10 pts) Compute the most general form for f given f' below. **Don't forget to add an arbitrary constant!**
   (a) $f'(x) = 2x^3 + 3x^{-3} + x^{1/2} - 10$
   (b) $f'(x) = 3\sin x + \sec^2 x$

3. (a) (10 pts) Find two numbers whose difference is 100 and whose product is a minimum. (**I will ask a different type of question here!**)
   (b) (5 pts) Use the first (or second) derivative test to show that the the number that you computed in (a) is does indeed minimize the product.

4. (15 pts) Let $f(x) = x^2 - x - 1$. Use Newton's method with $x_0 = 2$ to compute a root of f to five decimal places of accuracy (1.61803). Recall Newton's Method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

5. Let $f(x) = 2x^3 - 3x^2 - 36x + 6$.
   (a) (10 pts) Find the critical points **and** the inflection point of f.
   (b) (10 pts) Find the value(s) of x that satisfy the conclusions of the Mean Value Theorem on $[0, 1]$.

6. Let $f(x) = \frac{x}{x^2+9}$.
   (a) (5 pts) Find the critical points of f.
   (b) (10 pts) Find the intervals on which f is increasing and decreasing and determine the local maximum and minimum values of f.
   (c) (10 pts) Sketch a graph of f. **Label the local max and min values of f, the x and y intercept(s), and the horizontal asymptote.**

   Extra Credit: Assuming m and n are positive integers, find

$$\lim_{x\to-\infty} \frac{1 + 2x^n}{1 - 3x^m}.$$

   (**Hint:** Your answer will depend on whether $m < n$, $m = n$, or $m > n$.)