MATH 150: Answers to even numbered HW assignment problems

Section 1.1

# 4: (a) When $1000 is spent on advertising, the number of sales per month is 3500; (b) I; (c) the number of sales if no money is spent on advertising.

# 10: $f(5) = 4.1.$

# 12 (a) and (b):

![Expected return vs. Risk graph]

# 14: (a) At $t = 30$ minutes the temperature was $10^\circ$C. (b) Initial temperature of the object was $a^\circ$C. At time $t = b$ the object’s temperature is $0^\circ$C.

# 18: (a) III; (b) Vertical intercept represents the temperature of potato at $t = 0.$

# 20:

![Heart rate vs. time graph]

Section 1.2

# 2: Slope is $-3/2$, vertical intercept is 4.

# 8: $y = -7 + 3x.$

# 12: (a) $P = 30,700 + 850t$; (b) $P(10) = 39,200$. (c) Population will reach 45,000 in about 16.82 years ($t = 16.82$), or during the year 2016.
Section 1.3
# 2: concave up.
# 6: (a) intervals between $D$ and $E$, between $H$ and $I$; (b) intervals between $A$ and $B$, between $E$ and $F$; (c) intervals between $C$ and $D$, between $G$ and $H$; (d) intervals between $B$ and $C$, between $F$ and $G$;
# 18: (a) 16.4 billion $; (b) 3.28 billion $ per year; (c) $-6.0$ billion $ between 2000 and 2001.

Section 1.5
# 2: (a) initial amount = 100; growth; growth rate = 7 % = 0.07; (b) initial amount = 5.3; growth; growth rate = 5.4 % = 0.054; (c) initial amount = 3500; decay; decay rate = - 7 % = - 0.07; (b) initial amount = 12; decay; decay rate = -12 % = 0.12.
# 6: (a) $Q = 30 - 2t$,

(b) $Q = 30 \times (0.88)^t$, 

\begin{center}
\begin{tabular}{c|c|c}
\hline
$t$ (days) & 15 & \hline
\hline
$Q$ (grams) & 30 & \hline
\end{tabular}
\end{center}
# 10: (a) II; (b) I; (a) III; (b) V; IV shows exponential decay; VI shows linear decay.
# 18: (a) neither; (b) exponential: \( s(t) = 30.12 \times (0.6)^t \); (c) linear: \( g(u) = -1.5u + 27 \).
# 24: (a) The investment was worth $3486.78 after 10 years; (b) it will take about 11 years to get the investment back to $10,000.

Section 1.6
# 8: \( t = \ln 10 \approx 2.3 \).
# 22: (a) Town D is growing the fastest. (b) Town C is largest at \( t = 0 \). (c) Town B is decreasing in size.
# 24. \( P = 2 \times (1/\sqrt{e})^t \approx 2 \times (0.6065)^t \).

Section 1.8
# 2: (a) \( f(t + 1) = (t + 1)^2 + 1 = t^2 + 2t + 2 \), (b) \( f(t^2 + 1) = (t^2 + 1)^2 + 1 = t^4 + 2t^2 + 2 \), (c) \( f(2) = 5 \), (d) \( 2f(t) = 2t^2 + 2 \), (e) \([f(t)]^2 + 1 = t^4 + 2t^2 + 2 \).
# 8: (a) \( f(g(x)) = 2x^2 + 12x + 18 \), (b) \( g(f(x)) = 2x^2 + 3 \), (c) \( f(f(x)) = 8x^4 \).
# 12: (a) \( y = u^6 \), where \( u = 5t^2 - 2 \); (b) \( P = 12e^u \), where \( u = -0.6t \); (c) \( C = 12 \ln(u) \), where \( u = q^3 + 1 \).
# 30: (a) \( f(g(0)) = f(2) = 3 \); (b) \( f(g(1)) = f(3) = 4 \); (c) \( f(g(2)) = f(5) = 11 \); (d) \( g(f(2)) = g(3) = 8 \); (e) \( g(f(3)) = g(4) = 12 \).

Section 1.9
# 2: yes, \( y = 3x^{-2} \), \( k = 3 \), \( p = -2 \).
# 6: yes, \( y = (5/2)x^{-1/2} \), \( k = 5/2 \), \( p = -1/2 \).
# 14: \( E = kv^3 \), where \( k \) is a constant.

Review for Chapter 1
# 6: (b) 200; (c) 80; (d) \( 0 \leq 0 \leq 560 \); (e) decreasing; (f) concave down.
# 24: \( y = 0.4x + 2 \).
# 32: \( g(x) = 30.8 - 3.2x \) could be linear; \( h(x) = 15 \cdot 0.6^x \) could be exponential; \( f(x) \) is neither.

Section 2.1
# 4: slope -3: point F; slope -1: point C; slope 0: point E; slope 1/2: point A; slope 1: point B; slope 2: point D.
# 6:  (a) The average rate of change between \( x = 0 \) and \( x = 3 \) is greater than the average rate of change between \( x = 3 \) and \( x = 5 \) since slope of \( AB \) > slope of \( BC \)

(b) The function is increasing faster at \( x = 1 \) than at \( x = 4 \). Thus, instantaneous rate of change at \( x = 1 \) is greater than that at \( x = 4 \).

# 14:
\[
\begin{array}{cccccc}
  x & d & b & c & a & e \\
  f'(x) & 0 & 0.5 & 2 & -0.5 & -2 \\
\end{array}
\]

# 24:  (a) \( f(4) > f(3) \);  (b) \( f(2) - f(1) > f(3) - f(2) \);  (c)
\[
\frac{f(2) - f(1)}{2 - 1} > \frac{f(3) - f(1)}{3 - 1};
\]

(d) \( f'(1) > f'(4) \).
Section 2.2
# 10: IV.
# 14: (a) $f'(2) \approx 3$; (b) $f'(x)$ is positive for $0 < x < 4$ and is negative for $4 < x < 12$

Section 2.3
# 4: (a) $f(200) = 350$ means that it costs $350 to produce 200 gallons of ice cream; (b) $f'(200) = 1.4$ means that when the number of gallons produced is 200, it costs $1.4 to produce an additional gallon.
# 20: (a) $f(140) = 120$ means that a patient weighing 140 pounds should receive a dose of 120 mg; $f'(140) = 3$ tells us that if the weight of a patient increases by 1 pound (from 140 pounds), the dose should be increased by 3 mg; (b) $f(145) \approx 120 + 3 \times 5 = 150$ mg.
# 22: $f(22) \approx f(20) + f'(20) \cdot 2 = 345 + 6 \cdot 2 = 357$.

Section 2.4
# 2: $f'(x) > 0$, $f''(x) > 0$.
# 4: $f(x) < 0$, $f''(x) = 0$.
# 10: $f'(t) > 0$ on the intervals $0 < t < 0.4$ and $1.7 < t < 3.4$; $f'(t) < 0$ on the intervals $0.4 < t < 1.7$ and $3.4 < t < 4$; $f''(t) > 0$ on the interval $1 < t < 2.6$; $f''(t) < 0$ on the intervals $0 < t < 1$ and $2.6 < t < 4$.
# 20: (a) $f(x) < 0$ at $x_4$ and $x_5$; (b) $f'(x) < 0$ at $x_3$ and $x_4$; (c) $f(x)$ is decreasing at $x_3$ and $x_4$; (d) $f'(x)$ is decreasing at $x_2$ and $x_3$; (e) slope of $f(x)$ is positive at $x_1$, $x_2$ and $x_5$; (f) slope of $f(x)$ is increasing at $x_1$, $x_4$ and $x_5$.

Review for Chapter 2
# 10:

![Graph showing $f'(x)$]

# 18: $f(26) \approx f(25) + f'(25) \times 1 = 3.4$; $f(30) \approx f(25) + f'(25) \times 5 = 2.6$. 

5
#20:  (a) $f(1800) = 155$: consuming 1800 Calories per day results in a weight of 155 pounds;  $f'(2000) = 0$: consuming 2000 Calories per day causes neither weight gain nor loss;  (b) units of $dW/dc$ are pounds/(Calories/day).

Section: Focus on Theory (Chapter 2, pp. 135 – 140)

#10:  $0 \leq x \leq 2$: NO,  $0 \leq x \leq 0.5$: YES.

#16:  YES.

#24:  

$$f'(x) = \lim_{h \to 0} \frac{5(x + h) - 5x}{h} = \lim_{h \to 0} 5 = 5.$$  

#32:  

$$f'(x) = \lim_{h \to 0} \frac{2(x + h)^2 + (x + h) - 2x^2 - x}{h}$$  

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + h}{h} = \lim_{h \to 0} (4x + 2h + 1) = 4x + 1.$$  

Section 3.1

#4:  $y' = -12x^{-13}$.

#14:  $y' = 24t^2 - 8t + 12$.

#16:  $y' = -12x^3 - 12x^2 - 6$.

#24:  

$$y' = 2z - \frac{1}{2z^2}.$$  

Section 3.2

#6:  $f' = 5 \cdot 5^x \ln(5) + 6 \cdot 6^x \ln(6)$.

#16:  

$$y' = (\ln 10) \cdot 10^x - \frac{10}{x^2}.$$  

#18:  $D' = -1/p$.

#22:  $f' = Ae^t + B/t$.

#26:  $f(0) = 1040$ megawatts;  $f(15) = 1040 \cdot (1.3)^{15} = 53,233$ megawatts;  $f'(0) = 1040 \ln(1.3) = 273$ megawatts/year;  $f'(15) = 1040 \cdot \ln(1.3) \cdot (1.3)^{15} = 13,967$ megawatts/year.

Section 3.3

#6:  $w' = 15 \cdot (5r - 6)^2$.  

6
# 16: \[ f' = 30e^{5x} - 2xe^{-x^2} \].

# 22: \[ f' = \frac{2t}{t^2 + 1} \].

# 30: \[ y' = 2(5 + e^x)e^x \].

# 38: \( f(4) = 27e^{-0.14(4)} = 15.4 \) ng/ml; \( f'(4) = -3.78e^{-0.14(4)} = -2.16 \) ng/ml per hour.

Section 3.4

# 8: \( y' = e^t(t^2 + 2t + 3) \).

# 16: \[ f' = \frac{e^{-z}}{2\sqrt{z}} - \sqrt{z}e^{-z} \].

# 26: \[ y'(x) = \frac{e^x}{(1 + e^x)^2} \].

# 28: \[ y'(z) = \frac{z\ln z - 1 - z}{z(\ln z)^2} \].

Section 3.5

# 6: \( y' = 5\cos x - 5 \).

# 8: \( R' = 5\cos(5t) \).

# 14: \( y' = 12\cos(2t) - 4\sin(4t) \).

Review for Chapter 3

# 8: \( s' = 15t^2 - 2t + 20 \).

# 20: \( R' = 5(\sin t)^4 \cdot (\cos t) \).

# 36: \[ h' = \frac{2p}{(3 + 2p^2)^2} \].

Section 4.1

# 8: \( f'(x) = 3x^2 - 6 \), and thus, \( f'(x) = 0 \) at \( x = \pm\sqrt{2} \). \( f'(x) \) changes from positive to negative at \( x = -\sqrt{2} \), and so there is a local maximum at \( x = -\sqrt{2} \). \( f'(x) \) changes from negative to positive at \( x = +\sqrt{2} \), and so there is a local minimum at \( x = +\sqrt{2} \).
# 12: \( f'(x) = \ln x + 1 \), so \( f'(x) = 0 \) when \( \ln x = -1 \), that is, for \( x = e^{-1} \approx 0.37 \). This is the only place where \( f' \) changes sign. Since \( f''(1) > 0 \), the function \( f \) increases for \( 0 < x < e^{-1} \) and increases for \( x > e^{-1} \). Thus, we have local minimum at \( x = e^{-1} \).

**Section 4.2**

# 16: critical points at \( x = 1 \) (local min) and at \( x = 0 \) (neither min nor max); inflection points at \( x = 0 \) and at \( x = 2/3 \).

**Section 4.3**

# 22: Local and global maximum at \( x = 2 \).

# 30: 

\[ \frac{dE}{dF} = 0.25 - \frac{2 \cdot (1.7)}{F^3} = 0, \]
thus, 

\[ F = \left( \frac{2 \cdot (1.7)}{0.25} \right)^{1/3} = 2.4 \text{ hours.} \]

This gives a local and global minimum.

# 44: (a) At \( t = 0 \) we have \( q(0) = 0 \); (b) Maximum value occurs at \( t = \ln 2 = 0.69 \) (where \( q'(t) = 0 \)); maximum value is \( q(\ln 2) = 5 \text{ mg} \); (c) as \( t \to \infty \) we have \( q(t) \to 0 \).

**Section 4.4**

# 2: Profit function is positive for \( 5.5 < q < 12.5 \) (when \( R(q) > C(q) \)), and negative for \( 0 < q < 5.5, q > 12.5 \) (when \( R(q) < C(q) \)). Profit maximized when \( R(q) > C(q) \) and \( R'(q) = C'(q) \) which occurs at about \( q = 9.5 \).

# 16: Profit is maximized at \( q = 75 \); profit = \( R(75) - C(75) = 6875 \).

**Section 4.8**

# 6: (a) At peak concentration \( C'(t) = 0 \); corresponding \( t = 1/0.03 = 33.3 \text{ minutes} \); \( C(33.3) \approx 245 \text{ ng/ml} \); (b) after 15 min \( C(15) \approx 191 \text{ ng/ml} \); after one hour \( C(60) \approx 198 \text{ ng/ml} \).

**Review for Chapter 4**

# 8: (a) Increasing for all \( x \). (b) No max or min.

# 10: (a) Decreasing for \( x < -1 \); \( 0 < x < 1 \); increasing for \( x > 1 \); \( -1 < x < 0 \). (b) \( f(-1) \) and \( f(1) \) are local minima; \( f(0) \) is a local maximum.
Section 7.1
# 6:  \( t^8/8 + t^4/4 + C \).
# 14:  \( \frac{2}{3} z^{3/2} + C \).
# 18:  \ln |z| + C.
# 46:  \( \frac{e^{2t}}{2} + C \).
# 50:  \( 2x^4 + \ln |x| + C \).

Section 7.2
# 4:  \( e^{5t^2} + C \).
# 12:  \( \frac{1}{6} (x^2 + 3)^3 + C \).
# 20:  \( -\frac{1}{8} (\cos \theta + 5)^8 + C \).
# 34:  \( \frac{1}{2} \ln (y^2 + 4) + C \).
# 36:  \( 2e^{\sqrt{y}} + C \).
# 38:  \ln (2 + e^x) + C.

Section 7.3
# 6:  \[ \int_1^4 \frac{1}{\sqrt{x}} \, dx = 2. \]
# 10:  \[ \int_1^2 5t^3 \, dt = \frac{75}{4}. \]
# 22:  \[ \int_0^2 x(x^2 + 1)^2 \, dx = \frac{62}{3}. \]

Section 5.3
# 2:  Area = 8.
# 4: Negative.

# 6: Zero.

# 16: (a) 13; (b) −2; (c) 11; (d) 15.

**Section 5.4**

# 4: The change in velocity between times \( t = 0 \) and \( t = 6 \) hours; it is measured in km/hr.

# 12: 2627 acres.

**Section 5.5**

# 6: The fixed cost is $500. Total variable cost is \( \approx $866.7 \). Total cost is $500 + $866.7 = $1,366.7.

**Review for Chapter 5**

# 10:

\[ \int_{1}^{5} (x^2 + 1)dx = \left( \frac{x^3}{3} + x \right) \bigg|_{1}^{5} \approx 45.33. \]

# 14:

\[ \int_{1}^{3} (z + 1/z)dz = \left( \frac{z^2}{2} + \ln(z) \right) \bigg|_{1}^{3} \approx 5.10. \]

# 32: Since \( f(x) = \sqrt{1+x^4} \) is increasing for \( 0 \leq x \leq 2 \), we have

\[ \int_{0}^{2} f(0)dx \leq \int_{0}^{2} f(x)dx \leq \int_{0}^{2} f(2)dx, \]

so that

\[ 2 = \int_{0}^{2} dx \leq \int_{0}^{2} f(x)dx \leq \int_{0}^{2} 3dx = 6. \]

# 36: Total cost is 4,250,000 riyals.
Section 6.1

# 2: 
Average value \[= \frac{1}{3} \int_0^3 f(x)dx = 8.\]

# 4: Average value = 2.
# 6: Average value \(\approx 2202.55.\)
# 12(a): Average inventory \(\approx 527.25.\)
# 22: \((c) < (a) < (b).\)

Section 7.4

# 6:

\[F(0) = 1, \quad F(0) = 0, \quad F(1) = -1, \quad F(3) = 7, \quad F(1) = -1, \quad F(4) = 5.\]

Review for Chapter 7

# 6: 
\(\ln |x| - \frac{1}{x} - \frac{1}{2x^2} + C.\)

# 20: \(3 \sin(x) + 7 \cos(x) + C.\)
# 36: \(\sqrt{x^2 + 4} + C.\)

Section 10.1

# 2: (a) (III); (b) (V); (c) (I); (d) (II); (e) (IV).
# 8: Rate of change of \(B = \text{Rate in} - \text{Rate out}:\)

\[\frac{dB}{dt} = 0.04B - 200.\]

Section 10.2

# 16: E.
# 18: A.
# 22: (a) II; (b) I.

**Section 10.4**

# 2: \( w = 30e^{3r} \).
# 4: \( Q = 50e^{t/5} \).
# 6: \( p = 164.87e^{-0.1q} \).

**Section 10.5**

# 4: \( P = 104e^t - 4 \).
# 6: \( Q = 400 - 350e^{0.3t} \).