M514, Spring 2010: HW # 2

Read Section 2.1 of the book as well as a part of Section 2.2.3 related to formulation of equations of chemical kinetics.

Exercises:

In all the problems formulated below \( \varepsilon \) is a small parameter: \( 0 < \varepsilon \ll 1 \).

1. Define the asymptotic solution up to the terms of the order \( O(\varepsilon) \) for

\[
\varepsilon \frac{dz}{dt} = -z + y + \varepsilon t^2, \\
y' = z + y, \\
z(0) = 0, \quad y(0) = 1, \quad t \in [0, 1].
\]

2. Construct the zeroth order approximation of the solution of

\[
\varepsilon \frac{dz}{dt} = (y^2 - z^2) \cos t, \\
y' = z + y, \\
z(0) = 0, \quad y(0) = 1, \quad t \in [0, 1].
\]

3. Consider three chemical reactions \( 2A_1 \xrightarrow{k_1^+} A_2, \quad 2A_2 \xrightarrow{k} A_3 \) involving three substances.

a). Write down the system of equations for concentrations \( x_1, x_2 \) and \( x_3 \) of the substances \( A_1, A_2 \) and \( A_3 \), respectively, describing these reactions. Notice that differential equations for \( x_1 \) and \( x_2 \) do not contain \( x_3 \) and, therefore, could be considered independently of the equation for \( x_3 \).

b). Assume that \( k_1^+ = k_1^- = \frac{1}{\varepsilon} >> 1 \), i.e. the forward and reverse reactions involving substance \( A_1 \) are fast, and \( k = 1 \), i.e. the last reaction is comparatively slow. Find the zeroth order approximation of the solution subject to initial conditions \( x_1(0) = 1 \) and \( x_2(0) = 0 \), \( x_3(0) = 0 \).